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EFFECT OF AXIAL CURVATURE ON AERODYNAMIC CHARACTERISTICS OF PLANAR CHANNELS

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The effect of axial curvature on average characteristics of a turbulent flow of incompressible liquid in the gap between two coaxial cylinders is studied.

Calculation of aerodynamic characteristics of channels with cylindrical walls for laminar flow was performed in [1]. Experimental studies of turbulent flow in curvilinear channels were performed in [2-5]. Both averaged characteristics [2-5] and turbulent flow structure [3] were studied. However experimental data have been obtained only for relatively slightly curved channels ($r_m > 4.5H$). At the same time, technological applications employ highly curved channels (see, e.g., [6]).

The present study is a systematic calculation of the effect of axial curvature on the averaged characteristics of a turbulent flow in a channel with cylindrical walls, these characteristics being the velocity profile, tangent stress coefficient, and resistance coefficient, for various Reynolds numbers Re. For the sake of definiteness we will consider a stabilized flow, in which the velocity does not change along the axial coordinate. Such a flow is defined by one geometric parameter, the relative curvature r_m/H (or r_1/r_2), and in the case of an incompressible liquid, by one regime parameter, the Reynolds number Re.

It is known that curvature has a significant effect on turbulent exchange. At the present time various methods have been proposed to consider the effect of curvature on turbulent friction. In [7] an analysis and calculated comparison was made of various semiempirical turbulence hypotheses for a stabilized flow in a round channel. It was shown that the generalized theory of the Prandtl displacement path

$$\tau_t = \rho l^2 \alpha^2 |\theta| \theta, \tag{1}$$

$$\theta = \frac{du}{dr} - \frac{u}{r} \tag{2}$$

produces satisfactory results on the whole. Prandtl's formula may be obtained from the turbulent energy balance equation, if we neglect convective (absent in stabilized flows) and diffusion terms and also take $\nu_t = Cle^{-1/2}$. Since the term $\tau_t\theta$ is written exactly, this expression considers the major effect of curvature, i.e., additional generation of turbulent energy at the concave wall, and suppression of the same at the convex wall.

As is well-known (see, e.g., [9]) the displacement path length is weakly dependent on flow conditions: for a boundary layer on a plane plate and for developed flow in a tube the expressions for displacement path length practically coincide, and have little effect on Re and the axial pressure gradient. Also, as follows from physical considerations [9] and directly from measurement [3], the integral scale of turbulence (and consequently, the displacement path length) increases somewhat at the concave wall and decreases somewhat at the convex wall. However at present there are no systematic experimental data on the effect of curvature on the turbulence scale.

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In many studies of jets along a wall and boundary layers on curvilinear surfaces, Bradshaw's correction for displacement path length [10] is used, based on the analogy between curvilinear flows and temperature stratification of a medium, with use of experimental data for an atmospheric boundary layer. This correction, formally related to the displacement path length, is used with the equations of motion and turbulence model describing rectilinear flow and considers mainly the additional generation of turbulent energy in curvilinear flow. The presence of an empirical constant in Bradshaw's correction permits improvement of the correlation with experiment in each concrete case by suitable selection. However its use in calculating flow in a channel is not desirable. As was shown in [7], with the best choice of empirical constant ($\beta = 1$ with consideration of corresponding terms in the motion and turbulence equations, not $\beta = 7$ and 4, as Bradshaw recommends) the Bradshaw correction improves agreement with experiment only insignificantly. This is true mainly because the analogy between curvilinear flow in a channel and stratification in the boundary layer of the earth is valid only at low curvatures and near the wall; it is not useful for describing the flow in the central portion of the channel.

We also note that introduction of turbulent viscosity and ignoring turbulent diffusion (in particular, coarse scale diffusion) do not permit consideration of that peculiarity of curvilinear flow in a channel that the zero point and the point where $\theta = 0$ do not coincide. Even so, the divergence between calculation and experiment obtained with this approach is not large. Consideration of the ignored factors requires transition to more complex hypotheses, which, in view of the absence of necessary data, seems premature.

In the present study, to describe turbulent friction we will use Prandtl's generalized formula, Eq. (1), together with the complete equation of motion.

The equation of motion for a stabilized flow of viscous incompressible liquid in a plane circular channel in terms of shears has the form

$$\frac{d}{dr}(\tau r^2) = r\frac{\partial p}{\partial \varphi}, \tag{3}$$

where

$$\tau = \tau_l + \tau_t = \rho\nu\theta + \rho l^2\alpha^2|\theta|\theta. \tag{4}$$

The displacement path length is then specified by the Nikuradze—Prandtl formula with the Van Driest correction at the wall

$$\frac{l}{\delta} = \left[0.14 - 0.08\left(1 - \frac{y}{\delta}\right)^2 - 0.06\left(1 - \frac{y}{\delta}\right)^4\right]\alpha, \tag{5}$$

$$\alpha = 1 - \exp\left(-\frac{y\sqrt{\tau/\rho}}{26\nu}\right), \tag{6}$$

where δ is taken as the distance from the wall to the point of zero shear stress.

We integrate Eq. (3), considering that $\partial p/\partial\varphi$ is constant along the radius

$$\tau = \frac{1}{2}\frac{\partial p}{\partial \varphi}\left(1 - \frac{r_0^2}{r^2}\right), \tag{7}$$

while for $r = r_0$, $\tau = 0$. It is convenient to express $\partial p/\partial\varphi$ and r_0 in terms of the shear stress on the walls

$$r_0 = r_1\sqrt{1 - 2\frac{\tau_1}{\partial p/\partial \varphi}}; \tag{8}$$

$$\frac{\partial p}{\partial \varphi} = 2\frac{\tau_2 - \tau_1(r_1/r_2)^2}{1 - (r_1/r_2)^2}. \tag{9}$$

The system of Eqs. (2), (4)-(9) together with boundary adhesion conditions ($u = 0$ at $r = r_1$ and $r = r_2$) is a closed system which can be solved numerically in the following manner.

The initial data for the solution are the ratio of the radii r_1/r_2 and the Reynolds number Re.

If τ_1 and τ_2 are known, Eqs. (9), (8), (7) are used to determine the friction distribution. With consideration of Eqs. (5) and (6), we use Eq. (4) to find the θ distribution.

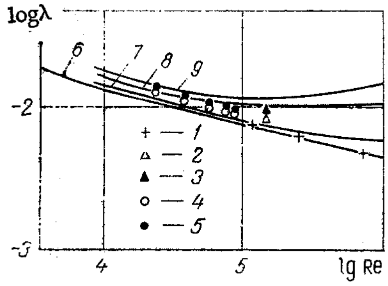

Fig. 1

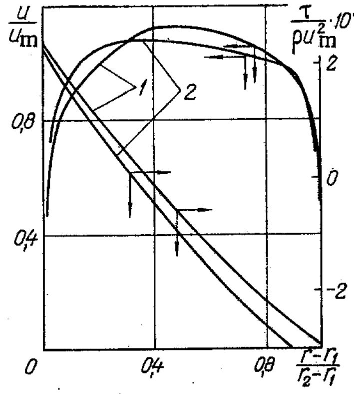

Fig. 2

Fig. 1. Resistance laws: 1-5) experiment: 1) [11]; 2,3) [3]; 4,5) [2]; 6-9) computation: 1,2,4,6) rectilinear channel; 3,5,7-9) curvilinear channel: 3) $r_m/H = 9.5$; 5,7) 4.5; 8) 2; 9) 1.5.

Fig. 2. Calculated (1) and experimental [2] (2) velocity profiles for curvilinear channel with $r_m/H = 4.5$ at Re = $2.5 \cdot 10^4$.

With a known θ distribution, we use Eq. (2) with the adhesion condition on one wall u = 0 at $r = r_1$ to find the velocity distribution.

Since τ_1 and τ_2 are not known beforehand, their values will be refined in the computation process. Specifying a τ_1 value, the value of τ_2 is chosen by the "target" method such that the adhesion condition is satisfied on the second wall. Similarly, the "target" method is used to select an appropriate τ_1 value to obtain a specified Re.

Calculations were performed with a constant step in r. The number of points computed was chosen so that there were no less than two points in the viscous sublayer, which ensured an accuracy of 1-2% in determination of λ and τ_2/τ_1.

Figure 1a compares the resistance laws for a plane rectilinear channel, obtained by calculation, with the experimental data of Cont-Bello [11], Wattendorf [2], and Eskinazi and Yeh [3]. The divergence between experiment and calculation does not exceed the scattering of the experimental data. There is especially good agreement with the latest, and apparently most accurate, data of Cont-Bello.

According to Wattendorf's data, the resistance coefficient of a curvilinear channel with $r_m/H = 4.5$ increases by 5-7% compared to that of a rectilinear channel (Fig. 1). The same result was achieved by computation (Fig. 3).

Figure 2 shows calculated [2] and experimental velocity profiles and shear stress. The divergence between calculated and experimental τ profiles does not exceed 7% of the maximum shear stress value τ_{max} (Fig. 2). The tangent stress distribution in the case under consideration is characterized completely by two parameters: the position r_0 of the point where τ = 0 (or the ratio of the frictions at the walls), and the resistance coefficient λ, which determines the slope of the curve τ(r). The slope of the calculated curve agrees well with that of experiment. The position of the zero-friction point as determined experimentally is displaced toward the convex boundary compared to the calculated value. This is due to underestimation of the influence of curvature in the calculation caused by ignoring its effect on displacement path length.

In developed flow within a channel Eq. (3) does not relate the velocity distribution to the shear stress as in the case of a boundary layer. (The boundary layer equation $dp^*/ds = d\tau/dn$ with consideration of the fact that dp/dn = 0 and outside the boundary layer λ = 0 uniquely relates the velocity profile and the friction.) Thus for the case under consideration it is necessary to know both the shear stress profile and the velocity profile to completely define the characteristics of the averaged flow. On the whole the calculation does properly consider the effect of curvature on the form of the velocity profile, which manifests itself in a change in relative position of the points where u, ur, and u/r are maximal. Divergence between calculated and experimental velocity profiles does not exceed 7% (Fig. 2).

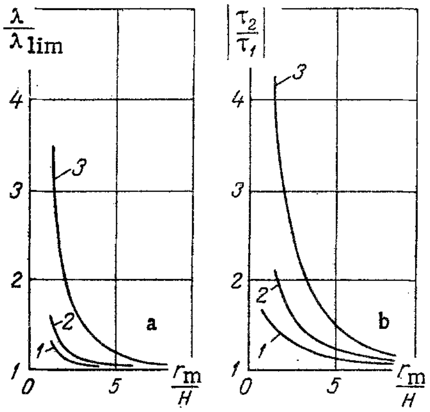

Fig. 3. Effect of channel curvature on resistance coefficient (a) and ratio of shear stress on walls (b) for various Re: 1) Re = 10^4; 2) 10^5; 3) 10^6.

Results of a numerical calculation of the effect of curvature on resistance coefficient and shear stress ratio on the walls at various Reynolds number Re are presented in Fig. 3, while Fig. 4 shows the effect of r_m/H on u and τ profiles.

Initially, with a marked decrease in radius of curvature the resistance coefficient changes only slightly: the increase in turbulent exchange on the one wall is compensated to a significant extent by a decrease upon the other. A sharp increase in resistance occurs at midline radii smaller than 2-3 channel heights. This is caused by a disruption of flow symmetry (Fig. 4). Such a dependence of λ on r_m/H may be related to the form of the τ profile. Since in a curvilinear channel turbulent exchange in the boundary layer on the concave side is greater than on the convex side, the shear stress on the concave side $|\tau_2|$ is greater than that on the convex τ_1, and the position of the zero shear point is thus shifted toward the convex side with increasing curvature. But at low curvatures such a displacement can occur without change in the slope of the profile, while in highly curved channels an increase in the slope of the profile (and thus, in the resistance coefficient λ) is unavoidable, since the shear on the wall cannot go to zero.

With increase in channel curvature the slope of the velocity profile at the concave wall (Fig. 4) increases with increase in $|\tau_2|$, while it decreases at the convex wall due to decrease of τ_1. In slightly curved channels ($r_m/H > 3$) the velocity maximum is located somewhat closer to the convex boundary. With increase in curvature the flow is "pressed" toward the concave boundary; at $r_m/H = 1.5$ (Re = 10^5) the velocity maximum is shifted to a position $(r - r_1)/H \approx 0.75$. The cause of this shift in the velocity maximum is the following. Since in slightly curved channels u/r is small, then the points at which θ = 0 (together with τ = 0) and the points where du/dr = 0 are close to each other, and since the position r_0 of zero shear stress is shifted toward the convex boundary, the point of maximum velocity is also shifted toward that side. At low r_m/H, u/r is quite high, and since at the point r_0 du/dr = u/r, du/dr is also quite high, so that the decrease in du/dr to zero occurs in a further removed region.

Axial curvature changes the character of the effect of Re on the resistance coefficient.

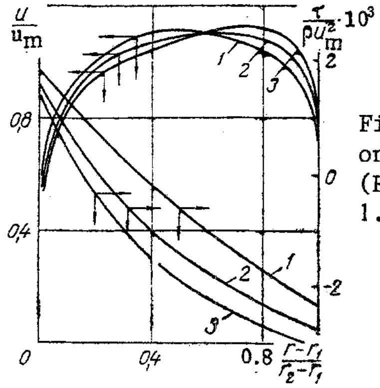

Fig. 4. Effect of channel curvature on velocity and shear stress profiles (Re = 10^5): 1) r_m/H = 4.5; 2) 2.0; 3) 1.5.

While in a straight channel the resistance coefficient falls with increase in Re (in the postcritical range), in highly curved channels the coefficient may increase beginning at a certain value of Re (Fig. 1). This behavior of the resistance law is related to the fact that the effect of curvature is intensified with increased Re (Figs. 3, 4). With increase in Re the shear stress on the concave wall increases due to intensification of turbulent exchange near that wall, while at the convex wall (near which turbulent pulsations are suppressed) the flow at sufficiently high curvature may remain laminar so that the shear stress then remains close to zero. This leads to sharp skewing of the τ profile (Figs. 3, 4): in the case of laminar flow $|\tau_2/\tau_1|$ is close to (only slightly less than) unity, while at $Re = 10^6$ and $r_m/H = 1.5$ this ratio becomes greater than 4. Thus the resistance coefficient λ increases (Fig. 3).

NOTATION

e, turbulent energy; $H = r_2 - r_1$, channel height; l, displacement path length; p, pressure; r, radius; $Re = u_m H/\nu$; u, velocity; $u_m = \int_{r_1}^{r_2} u dr/H$, mean velocity; y, distance from wall; α, Van Driest's correction; $\theta = du/dr - u/r$, angular deformation rate; $\lambda = -(\partial p/\partial \varphi)/(r_m 0.5 \rho u_m^2/H)$, resistance coefficient; ν, kinematic viscosity; ρ, density; τ, shear stress; φ, angular coordinate. Indices: l, laminar; t, turbulent; 0 corresponds to $\tau = 0$; 1, convex wall; 2, concave wall.

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